GRS 80 – the new height system

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GRS 80 — the new height system*)

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Abstract:

A model for combining satellite and terrestrial height data is presented. Since relative geoidal undulations can be obtained with an accuracy compatible with relative heights from levelling or relative ellipsoidal heights from satellite observations, a common adjustment can be done to establish the relations between the different heigth systems.

Introduction

Heights are a problem to today's geodesy. How can we establish a height system that has an accuracy compatible with the accuracy of the future space geodesy? During the last 100 years we have perfected the classical method of heighting, levelling in combination with gravity observations, and there we can see that the potential for more accuracy is exausted. We have reached the limit because of a fundamental weakness in the mathematical model.

The solution to the problem is obvious. We must introduce a system of ellipsoidal heights. The technical solution is straightforward, and the real problem may be the training of the users. The group of geodesists that now maintains the classical method of heighting, levelling and geoid determination, must now learn to accept ellipsoidal heights as the primary height system. Orthometric heights with less but sufficient accuracy can be derived from the system of precise ellipsoidal heights.

Datum from space

Satellite geodesy has had its own reference systems from the beginning. Apart from being geocentric from gravitational considerations, these satellite systems originally had little in common with terrestrial reference systems. However, the application of satellite tracking and positioning methods soon established relationships between satellite reference systems and global terrestrial reference systems.

The implication is that satellite positioning methods can now be used to position geodetic stations in a basic terrestrial reference system, such as the Average Terrestrial System (ATS). Also, this means that a local geodetic reference system can now be defined explicitly in the terrestrial system using those satellite positioning methods (e. g., Kouba, 1976, 1978).

The World Geodetic System 1984 (WGS 84) datum has been used for GPS since the beginning of 1987. It is now also used as datum for the TRANSIT precise ephemeris. It is the newest version of a series of satellite datums that has been established for US satellite navigation systems. Except for a small but greatly annoying difference in the ellipsoid flattening, the WGS 84 datum has adopted the parameters of the Geodetic Reference System 1980 (GRS 80).

WGS 84 is at present a good realization of ATS. Its origin is at the centre of gravity of the earth. Its Z-axis is oriented towards the Conventional International Origin (CIO) as defined by the International Polar Motion Service. The X-axis is oriented in the direction of the Greenwich Mean Astronomical Meridian as defined by the Bureau International de L'Heure (BIH). Because of the time variation in the dynamics of the Earth, average values over a certain period of time are defined as reference values. In the definition of ATS, the period 1900—1905 is used. The polar motion and longitude variations can therefore be reduced to those reference values with the appropriate corrections applied to geodetic positions.

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The different space systems such as TRANSIT, GPS, Very Long Baseline Interferometry (VLBI), Lunar Laser Ranging and Deep Space Networks are being compared and combined to provide more accurate positional and directional information about geodetic stations and datums.

Fiducial networks

Fig. 1 shows the result of 2,5 years of VLBI measurements for the 5600 km long baseline Westford-Onsala. The RMS scatter about a straight line fit through the length determinations is 30 mm. Similar accuracies are achieved for other baselines of a VLBI network spanning the world. Comparison of VLBI derived values for polar motion and values derived by Satellite Laser Ranging (SLR) shows an RMS difference of only 60 mm in X- and Y-components during 1984 (Carter et al, 1985).

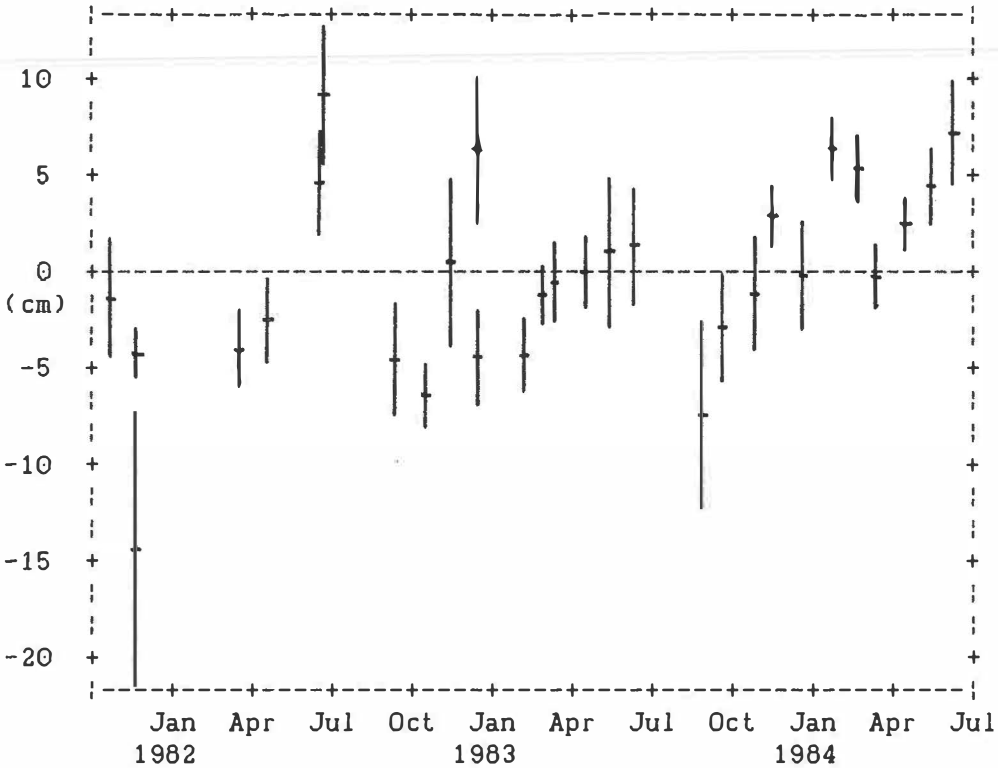


Fig. 1. Westford-Onsala VLBI length determinations, plotted as residuals about their weighted mean

The precise network of VLBI and SLR stations can now be densified by using the fiducial network method. GPS data collected at the VLBI and SLR stations, called fiducial sites, provide information essential to precise satellite orbit determination. Simultaneous observations at new stations then allow us to get precise ATS in areas not easily accessible to VLBI and SLR systems. Today the use of standard GPS receivers and the fiducial network method, can achieve 0.1 ppm in the horizontal components.

Heights from GPS

By using satellite observations we can get a network of stations that has coordinates in three dimensions. The heights derived from satellite observations are ellipsoidal heights (h), and as such they cannot be included directly in our network of levelled heights (H). However, such ellipsoidal heights are needed for control and strenghtening of existing levelled height networks. The two types of height are related via the geoidal undulation (N), by the formula

$$h = H + N \tag{1}$$

or, more correctly, by relative heights

$$d\,h = d\,H + d\,N \tag{2}$$

Equations (1) and (2) show that knowledge of the anomalous gravity potential is required to connect the height systems h and H. Fortunately, the three parts of equation (2) can all be determined with an accuracy that is of the same order of magnitude (Sideris, Schwarz, 1986).

In Scandinavia we can now use the new Nordic geoid (Tscherning, Forsberg, 1986) to derive geoidal undulation differences (dN). The deflections of the vertical can be predicted by that geopotential model at the one second of arc level, corresponding to a geoid slope of 5 mm per km, or 5 ppm for dN.

The levelling network of Norway has closure errors that correspond to an RMS error of 3—4 mm per km, which means that the accuracy of dH is 3—4 ppm.

Relative ellipsoidal heights (dh) from GPS, have accuracy of about 2—3 ppm, and will be improved in the future by using the fiducial method.

However, even if equation (2) is valid, this may not be the case for equation (1). Before we can use equation (1) for a common adjustment of levelled heights and ellipsoidal heights, we must do some preliminary research to see if the local network of levelled heights has a bias.

Accuracy of levelled heights

The level is a very precise instrument, and has been useful for measuring relative heights since the beginning of civilization. Such levelled heights were needed for planning and building of irrigation, and for transportation channels.

However, for fixing of a height datum, the heights above sea level do not have a precise definition. We cannot get a precise measure of mean sea level. By the use of tide gauges, the orthometric height system is tied to the local Sea Surface Topography (SST) at the coast. SST has amplitudes of 1—2 m, and thus we find closure errors when tide gauges at some distance are connected by precise levelling.

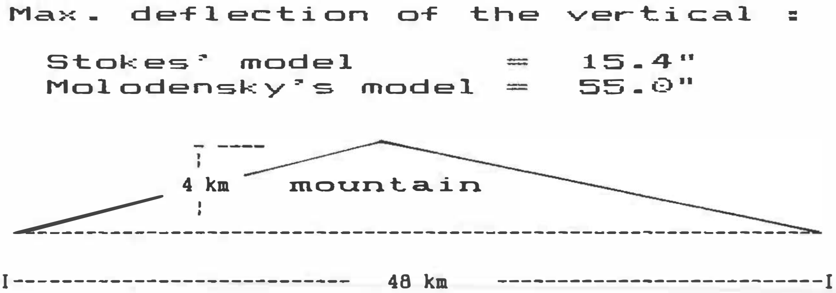
Even if we could measure mean sea level with the necessary precision, we are unable to get precise orthometric heights from these measurements, because we do not have a precise scale for the observations. By integration of levelled height difference (dh) and observed surface gravity (g), we can get precise difference of potential (dW) by the equation

$$dW = W_B - W_A = -\int_A^B g\,dh \approx -\sum_{i=1}^{n} g_i\,dh_i \tag{3}$$

In order to scale the observed potential difference into height difference, we need observations of gravity along the plumb line, which we cannot get.

The determination of the position and shape of the zero potential layer is called the solution of the free boundary value problem of physical geodesy. The solution of this problem has engaged many geodesists during the last century, but with limited success (Bjerhammar, 1967). The theory of Molodensky has shown that this problem has no unique solution. By introducing a slightly modified geoid, the quasigeoid, Molodensky found that we can get a unique solution for the height above the ellipsoid.

Fig. 2 from (Bjerhammar, 1967) is an example where the deviation of the vertical is computed for a mountain of height 4 km and base diameter 48 km. In flat low land any model can be used, but this example shows that in mountain areas the difference between the classical method, and the rigorous mathematical model of Molodensky becomes too big.



Max. deflection of the vertical :

Stokes' model = 15.4"
Molodensky's model = 55.0"

Fig. 2. Comparison of Stokes formulae (the classical method), and the method of Molodensky, for determination of the deviation of the vertical under a mountain of height 4 km

Today it is no big loss for geodesy to abandon the concept of the geoid. We should not forget that the geoid was a tool only, to be used for the production of precise coordinates on the surface of the Earth. Today we can get precise coordinates from GPS, and then we no longer need a precise geoid for that purpose.

The engineer who plans hydroelectric power plants or irrigation schemes, is interested in levelled heights, and may not be happy if the geodesist can only offer ellipsoidal heights. He will have no problems using the equation (2) for the combination of ellipsoidal heights and levelled heights, because he is working within a limited area.

However, the hydraulic engineer who needs the utmost accuracy, cannot use orthometric heights. Instead he should use dynamic heights. If we take as an example a tunnel of length 30 km in the north-south direction, at height 1000 m in the mountains of Norway, the orthometric heights of a level surface will differ by 2 cm at the ends of the tunnel, because of the orthometric correction. Most often the engineer does not need such high accuracy.

The reformation of geodesy

The precise levelling lines that cross the continents, were not made for the hydraulic engineers. They were made for the scientists who wanted to map the globe with the utmost precision. As discussed above, the classical concept of the geoid was a less successful tool for this work. Fig. 1 shows that today the geodesist can map the world by using space methods, without the use of a detailed geoid.

Fig. 3 shows a profile of the 500 km long levelling line between the two biggest cities of Norway. We have drawn into the profile an example of a fiducial network where two GPS satellites are shown, and we have included a levelling line of the classical type. The terrestrial line consists of 5000 pairs of 50 m long foresight/backsight rays, and we must measure them twice. At the scale of 1:250000000, these foresight/backsight set-ups cannot be seen in the drawing, because they follow the terrain at height 1.5 m.

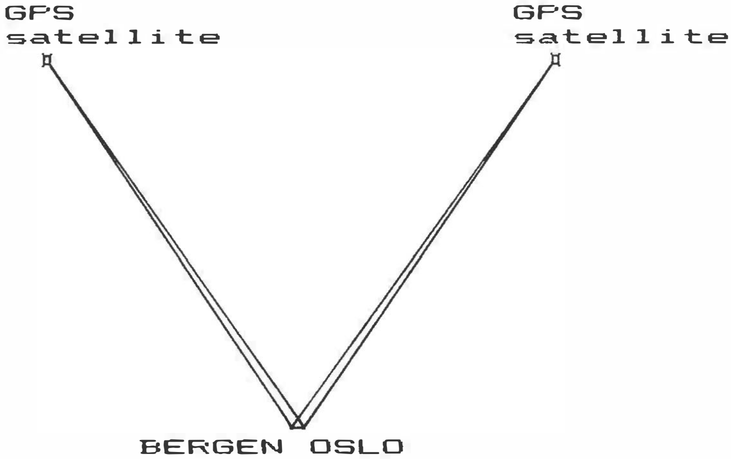


Fig. 3. Terrestrial levelling and GPS levelling between Bergen and Oslo

At this height above the ground, gravity observations or a detailed geopotential model is needed if we want high precision from the classical method of levelling. A geopotential model is also needed for the fiducial network, indirectly, for computation of the satellite orbits. However, for orbit computation at height 20.000 km above the surface of the earth, we do not need a detailed geopotential model. This example illustrates the big difference between the two methods of levelling.

With the classical method we combine differently oriented heigth differences and gravity, to get geopotential numbers, using equation (3). Each observed heigth difference of a levelling line has its own local reference frame, as the height is measured along the local plumb line.

By using GPS we get a single vector as the observation. This vector can be adjusted together with other observations in a common adjustment, as explained in (Hirvonen, 1962). Other pioneers have now started to apply those ideas (Vincenty, 1981):

"... in my opinion geodesy can do without the geodetic line without missing it at all. A geodetic line is a fiction, and so is its length and azimuth. On the other hand, the straight line in space between two points (not between their projection on the ellipsoid) is something real. Now, refer to my article in Bull. Geod. 54/1, 1980, on the height controlled system. It does not contain the notion of a geodetic line, but will be used in principle for the readjustment of North American Networks in 1983. There will be some modifications (e. g. geodetic horizon instead of astronomic), but the geodetic line will not appear anywhere in the mathematical model."

Combination of height systems

The network of levelled heights is still useful for the mapping of the globe. The levelling networks cannot span the oceans, but a coarse net of ellipsoidal heights from satellite data can be used to tie the various local levelled height datums into a global one. The datum of the global height system will be fixed by space methods such as VLBI. The levelled heights will be used for densification, and for determination of transformation parameters between the different height systems. As a first approximation for this transformation, we can make use of the computed geoid of the area. As mentioned above, the Nordic geoid can predict the vertical at the one second of arc level. This geoid must be published in a suitable digital form. Then the digital model of the geoid can be used by everyone as a definition of the geoid, free of error. Any error in the equation (1) can be modelled as an error in the orthometric height (H) by adding a correction term (H0)

$$h = H + H0 + N \qquad (4)$$

Once we have a good value for H0, the combination of datum bias (H0) and geoid undulation (N)

$$H = h - (H0 + N) \qquad (5)$$

can be used as the official transformation for that area, to derive orthometric heights from precise ellipsoidal heights. We need one definition of H0 for each existing local orthometric height datum. As an example, the city Bodø has 5 different height datums (NOU 1984: 4), and then we need 5 different sets of H0 in order to get a common height network for that area. If requirements for accuracy are high, it may be necessary to express H0 as a bilinear polynomial in X and Y.

We must accept that the adopted value for H0 and N can be changed in the future, if an improved geopotential model becomes available. In the future we can use GPS for positioning of gravity observations, and then the free boundary value problem of physical geodesy becomes a fixed boundary value problem, which allow for a more precise determination of the geoid (Sjøberg, 1986).

The orthometric heights will be used for engineering work only. We must be aware that for most engineering work, the ellipsoidal heights can be just as suitable as the present system of orthometric heights. In the lowland where most building activities take place, the deflections of the vertical stays below 10 seconds of arc, which is equivalent to a slope of 5 cm/km. Clearly, we would be unable to detect this small slope if our house had been levelled by using ellipsoidal heights. For scientific purposes such as the study of land uplift, we will be using ellipsoidal heights in a global datum such as GRS 80. Ellipsoidal heights can be transformed to any local datum once we have found suitable transformation parameters.

Conclusions

The classical height model relates levelled height differences and orthometric height by including gravity observations.

Due to the progress of satellite techniques, we can now make use of an alternative model that relates observed ellipsoidal height differences with ellipsoidal height. The future geodetic measuring methods will provide ellipsoidal heights as one dimension of earth-fixed three-dimensional coordinates.

The transition from the present system of orthometric heights to a future system of ellipsoidal heights is inevitable because of:

- the new type of observations
- the simple mathematical model that relates observations and parameters
- no need to collect additional physical data
- accuracy of model is not limited by hypotheses about crustal density.

When the scientist designs mathematical models for the description of the physical world, he is guided by the following fundamental law: The simplest model is the best model. Because of the criteria given above, we claim that the new model is better than the classical model (Schödlbauer, 1986).

An ellipsoidal height system is suitable for the study of land uplift, because of the simple and precise mathematical model of the height system. An ellipsoidal height system is also suitable for technical projects such as building of highways, and even for building of pipelines for gas, oil and water, as the slope of the geoid is less than 10 seconds of arc in flat terrain and in the highlands.

When dealing with some projects of hydraulic engineering and with hydrological and hydrographical investigations, orthometric heights or dynamic heights must be applied. A link between the different height systems is needed, in order to derive orthometric heights from precise ellipsoidal heights. The consistent use of a digital model for the geoid can provide the necessary link between the two systems of height.

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