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The Austrian Geoid – Recent Steps to a New Solution

Erhard Erker [1], Norbert Höggerl [2], Erich Imrek [3], Bernhard Hofmann-Wellenhof [4], Norbert Kühtreiber [5]

[1] *Bundesamt für Eich- und Vermessungswesen, Grundlagenvermessung, Schiffamtsgasse 1-3, A-1025 Wien*
[2] *Bundesamt für Eich- und Vermessungswesen, Grundlagenvermessung, Schiffamtsgasse 1-3, A-1025 Wien*
[3] *Bundesamt für Eich- und Vermessungswesen, Grundlagenvermessung, Schiffamtsgasse 1-3, A-1025 Wien*
[4] *Technische Universität Graz, Abteilung für Positionierung und Navigation, Steyrergasse 30, A-8010 Graz*
[5] *Technische Universität Graz, Abteilung für Positionierung und Navigation, Steyrergasse 30, A-8010 Graz*

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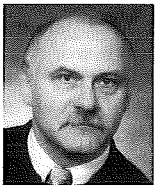
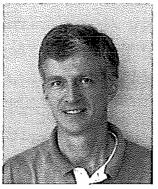
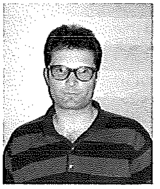
The Austrian Geoid – Recent Steps to a New Solution

Erhard Erker, Norbert Höggerl, Erich Imrek, Wien and Bernhard Hofmann-Wellenhof, Norbert Kühtreiber, Graz



Abstract

A refined version of the Austrian geoid with the working title „GEOID 2000" will be released after the IAG General Assembly in Sapporo. The project was worked out in a cooperation of the Federal Office of Metrology and Surveying and the Technical University of Graz, Institute of Geodesy. The territory of Austria serves as an ideal test area for the different computational methods concerning usability and accessible precision as well as for the compatibility of the available datasets. An overview of the computation process as well as the key figur es of the new geoid are discussed.

Zusammenfassung

Eine neue verfeinerte Version des Österreichischen Geoids mit dem Arbeitstitel „Geoid 2000" wird während der Generalversammlung der IAG in Sapporo präsentiert. Die Berechnungen wurden in einer Zusammenarbeit des Bundesamtes für Eich- und Vermessungswesen mit dem Institut für Geodäsie der Technischen Universität Graz durchgeführt.

Die unterschiedliche Topographie, die von den Alpen im Westen bis zu den großen Becken im Osten reicht, macht Österreich zu einem idealem Testgebiet für eine Geoidbestimmung. Dabei können die Anwendung und die Genauigkeit von Berechnungsmethoden einerseits und die Übereinstimmung verschiedenartiger Datensätze andererseits ideal untersucht werden.

Die vorliegende Arbeit beschreibt in den einleitenden Abschnitten die der Neuberechnung zugrunde liegenden Daten. Seit der letzten hochauflösenden Geoidberechnung 1987 wurden mehrere Datensätze stark verbessert sowie zusätzliche Daten erschlossen. So liegt nun ein umfassender Datensatz von Schwereanomalien vor. Für die Reduktion der Messgrößen wurde vom Bundesamt für Eich- und Vermessungswesen ein neues hochauflösendes Höhenmodell (44 m x 49 m) bereitgestellt. Weiters wurde ein homogener Datensatz von GPS Punkten verwendet, bei dem besonderer Wert auf die Genauigkeit der Höhenkomponente gelegt wurde. Für alle GPS Punkte liegen hochgenaue orthometrische Höhen, die in das europäische UELN-95/98, version 13, eingebunden sind, vor.

Für die Geoidberechnung kommt ein „Remove-Restore" Prozess zur Anwendung. Die Geoidhöhe wird mittels Kollokation aus Schwere- und Lotabweichungsdaten bestimmt. Um eine Aussage über die erreichbaren Genauigkeiten sowie die Möglichkeit der Kombination von Schwere und Lotabweichungen zu erhalten, wurden ein astrogeodätisches Geoid (nur Lotabweichungen), ein gravimetrisches Geoid (nur Schwereanomalien) und eine Kombinationslösung (Lotabweichungen und Schwereanomalien) bestimmt. Für die Kombinationslösung ist dabei eine eingehendere Untersuchung der Gewichte der Lotabweichungen im Verhältnis zu den Schwereanomalien notwendig.

Die Genauigkeit der einzelnen Lösungen wurde durch den Vergleich der resultierenden Geoidhöhen mit Geoidhöhen, die aus orthometrischen Höhen und ellipsoidischen Höhen (GPS) abgeleitet wurden, überprüft. Dazu wurden 3D-Koordinaten mithilfe der Geoidhöhen und orthometrischen Höhen abgeleitetet und in das Referenzsystem der 3D-Koordinaten aus GPS (System ETRF89) transformiert. Die Restklaffen der Transformation sind ein Maß für die Genauigkeit der Berechnungen. Für alle Geoidlösungen (astrogeodätische, gravimetrische und kombinierte Lösung) können die Restklaffen in einen Trend und Abweichungen davon aufgespalten werden. Grundsätzlich zeigt sich eine gute Übereinstimmung von astrogeodätischer und gravimetrischer Lösung. Das beste Resultat zeigt die komibinierte Lösung. Die Abweichungen der Restklaffen vom Trend liegen dabei im Mittel bei $\pm$ 1.4 cm und bestätigen die hohe Genauigkeit der Lösung.

1. Introduction

The first determination of a high resolution geoid in Austria was performed in 1987 using a set of more than 650 deflections of the vertical [1] [2]. In 1998, when a reasonable amount of gravity data for Austria and the neighboring countries was available, a gravimetric geoid was computed [3]. The results of a combination of both datasets, supported by GPS and leveling data, using the remove-restore technique and collocation, was presented at the International

Meeting of the Gravity and Geoid Commission (IGGC 2002) of IAG Section III in Thessaloniki in August 2002 [4]. By combining gravity data and deflections of the vertical the precision of the recent geoid solution was increased to the cm-level. Nevertheless, the use of only 50 GPS/leveling points with an inhomogeneous distribution was unsatisfactory. Therefore an additional GPS-campaign was initiated in October 2002. The improved GPS-leveling results and its comparison with the geoid solution will be discussed in the following.

2. Data

2.1. Digital Height and Density Model

All investigations are based on the same digital terrain model with a uniform resolution of 44 m x 49 m [5] and a constant density value of 2,67 g/cm^3.

2.2. Gravity

The basic gravity data set is a subset out of 86000 Austrian gravity observations and gravity material of the neighboring countries. In an inner zone (46.20° $\leq \phi \leq$ 49.21° and 9.25° $\leq \lambda \leq$ 17.25°) 5796 gravity data points were selected, approximately representing a grid with 6 km grid spacing. Additional data with the approximate grid spacing of 12 km were added in the outer zone (45.70° $\leq \phi \leq$ 49.70° and 8.50° $\leq \lambda \leq$ 18.20°). More details about the basic data set can be found in [4].

2.3. Deflections of the Vertical

The set of more than 650 deflections of the vertical used in the 1987 solution is used without modification in the following investigation.

2.4. GPS

As mentioned above, the last geoid determination [4] was supported by the use of 50 GPS/leveling points. The resulting geoid heights were fitted with the help of 12 selected points. The remaining 38 points were used for an external check.

In some parts unexplainable discrepancies between GPS/leveling and the three solutions (gravity, astro, combined) occurred. Therefore an additional GPS-campaign was initiated in October 2002, including some new leveling connections and an examination of the stability and identity of the observed points. As a result of this campaign a set of 102 points in the frame of AGREF/AREF (the Austrian GPS reference frame), which represents the Austrian densification of EUREF/ETRS89, was obtained.

All points were measured with at least 24-hour sessions during the last decade.

At 50% of the points observations of two independent sessions were available. The high precision of the up-component by repeated GPS-measurements is shown in Fig. 1. Height differences of 4 to 6 cm are due to early GPS measurements in the beginning 1990ies which may be disturbed by geometry, multipathing and troposphere. Additional measurements are planned.

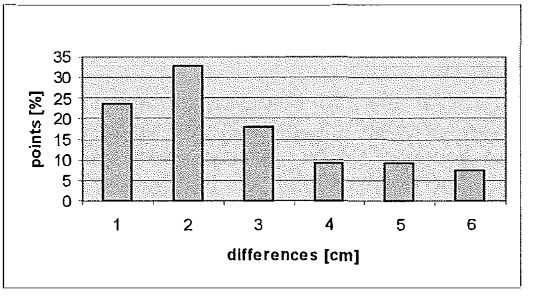


Fig. 1: Maximum height differences between double or multiple GPS-measurements

2.5. Leveling

2.5.1. Orthometric Heights

Orthometric heights refer to the geoid and are linked to ellipsoidal heights by the simple formula:

$$H_{ell} = N + H_{orth} \quad (1)$$

with

H_{ell} ellipsoidal height,
N geoid undulation,
H_{orth} orthometric height.

Orthometric heights can be derived with the formula

$$H_{orth} = C/g^* \quad (2)$$

where

$C = g\ .\ \Delta H$ ... geopotential number
g^* .... integrated mean value of the gravity between the surface and the zero-level
g .... gravity value at the surface point
ΔH .... levelled height difference

For more than ten years, the introduction of an orthometric height system in Austria has been under way. So far orthometric height values are available for all precise leveling points (more than 30.000).

2.5.2. The European Vertical Reference System EVRS

In order to establish a homogeneous height system for the whole European continent, the EUREF[1]) subcommission initiated a new adjustment of the 1st order leveling network of all European countries by the use of ΔC-values. For the Austrian part in this project the solution UELN-95/98 vers. 13 [6] was used as a basis for the computation of C-values for all precise leveling points. The adjustment of the ΔC values in the UELN was performed as an unconstrained adjustment linked to the reference point NAP (Normaal Amsterdams Peil). In this adjustment, the geopotential value of NAP is

$C_{NAP} = 0.$

As NAP refers to a local equipotential surface, a vertical offset to a global geoid has to be taken into account. So far, no world-wide geoid surface has been defined yet, that means that the above mentioned offset is unknown.

2.5.3. The estimation of the precision of orthometric heights

2.5.3.1. The geopotential number C

In Austria a gravity network based on 30 absolute gravity points and on about 700 first order points has been established. Along the precise leveling lines 23.000 gravity points have been measured. Additionally 15.000 gravity points, evenly spread over the territory of Austria, exist. By the use of a digital terrain model (~50 m x 50 m) the interpolation of gravity values for surface points with a precision better than 1 mgal (1.10^{-5} m/s^2) is possible. This is sufficient to achieve the same level of precision for the ΔC-values as for the ΔH values.

If the Austrian part of the adjustment of the UELN95/98 vers. 13 is considered, the C-values show a standard deviation of $m_{0/1} = \pm$ 0.8 kgalmm/km while for the C-values with reference to NAP $m_{cNAP} = \pm(10–12)$ kgalmm. If a central point in Austria is used as reference the standard deviation of the C-values is $m_{cA} = \pm(4–8)$ kgalmm (internal precision).

1) EUREF: IAG-Subcommission for Europe

2.5.3.2. The integrated mean values g of the gravity along the plumb line*

The computation of g*-values faces two problems:

- the determination of the integrated mean value of the gravity,
- the estimation of the influence of the varying mass density on the reduction process.

Intensive investigations have led to the following results. For the integration of the mean gravity value two cases are considered:

- For points with an altitude lower than 1400 m, the Kepler interpolation method is applied as it proved to give the best results. Consequently, the gravity for 3 points along the plumb line (with equal spacing) are used to calculate the mean value. For the weights the relationship 1-4-1 is chosen.
- For points above 1400 m altitude, Simpson's rule is applied. Five points along the plumb line (with equal spacing) are used, the weights for the gravity values of these points being 1-4-2-4-1.

The differences of the g* values computed by means of Kepler's method (or by Simpson's rule) in comparison with a g* value computed by the use of 20 intermediate points along the plumb line are smaller than 1.1 mgal [7].

The estimation of the influence of the varying density of masses can be summarised as follows:

An estimation of the influence of the varying density (2,8 g/cm^3 instead of 2.67 g/cm^3) on the orthometric height of a benchmark with an altitude of 2577m (Edelweißspitze/ Großglockner) which in fact is the highest precise leveling bench mark in Austria, shows a value up to 32 mm in maximum. On the other hand investigations done by Sünkel [8] show that larger density anomalies will be reduced by the fact that they are isostatically compensated.

It can be shown that for about 73% of the points used in this geoid investigation, the influence of the varying density is smaller than 8 mm (for a density variation of 15%); for 18% of the points the influence could be up to 15 mm.

Summarizing the above mentioned error influences on the determination of the orthometric heights, the following rough estimation can be given:

$m_{Horth} < 15$ mm (altitude < 1000 m),
$m_{Horth} < 20$ mm (altitude 1000 – 1500 m),
$m_{Horth} < 25$ mm (altitude 1500 – 2000 m).

3. The Geoid Computation

3.1. Remove-Restore

The geoid computation is done by the remove-restore procedure. The basic idea behind it is to take advantage of the fact that parts of the gravitational potential can be approximated by existing models. The long wavelength part is known through a given earth's gravitational model which is expressed in terms of a spherical harmonic expansion. The short wavelength part is a function of the mass (density) distribution of the topography and can be modeled by digital terrain and density models.

The remove-restore technique is applied in the following way. In the remove step residual gravity anomalies (gRES are computed by

$$\Delta g_{RES} = \Delta g - \Delta g_{EGM} - \Delta g_{DTM} \qquad (3)$$

The two effects removed from the gravity anomalies Δg are Δg_{EGM}, the long wavelength part of the gravity anomalies, and Δg_{DTM}, the short to medium wavelength part of the gravity anomalies. By this remove step we gain Δg_{RES} which represent a smooth field with only local-to-regional structures.

Here the adapted EGM96 [9] was used to compute the long wavelength part in the remove-restore procedure. For the short to medium wavelengths, a topographic isostatic reduction was performed using the adapted technique and a detailed height model with the resolution 11.25″ x 18.75″. For the isostatic model an Airy-Heiskanen approach with a standard constant density of 2.67 g/cm^3, a normal crustal thickness T of 30 km and a crust-mantle density contrast of 0.4 g/cm^3 was used. Table 1 shows the statistics for the reduction process.

Mgal	min	max	mean	std.dev.
Δg	–154.1	187.2	9.8	±42.2
$\Delta g - \Delta g_{EGM}$	–204.3	224.0	–1.1	±47.6
Δg_{RES}	–72.0	85.4	0.6	±23.6

Tab. 1: Gravity reduction using the standard density value of 2.67 g/cm^3 and the adapted geopotential model EGM96. Statistics are based on 5796 points (6 km x 6 km set).

After the remove-step the geoid heights N_{RES} are modeled from the residual gravity anomalies Δg_{RES}. In the following the estimation was done by collocation. Details on the used covariance function are given in chapter 3.2.

Finally the removed effects are restored again

$$N = N_{RES} + N_{EGM} + \delta N_{DTM} \qquad (4)$$

Here is the indirect effect which takes into account that removing the masses has changed the potential. N_{EGM} is computed using the spherical harmonic expansion of the earth's gravitational model.

This technique is commonly applied in local gravity field determination. Early computations done by this method are e.g. [2] and [11].

3.2. Covariance Function

The well-known Tscherning-Rapp covariance function model was used for the following LSC solutions. The global covariance function of the gravity anomalies $C_g(P,Q)$ given by Tscherning and Rapp ([12], p. 29) is written as

$$C_g(P,Q) = A\sum_{n=3}^{\infty}\frac{n-1}{(n-2)(n+b)}\, s^{n+2}P_n(\cos\psi) \qquad (5)$$

where $P_n(\cos\psi)$ denotes the Legendre polynomial of degree n, ψ is the spherical distance between P and Q and A, B and s are the model parameters. A closed expression for (Equ. 5) is available in (ibid., p. 45).

The local covariance function of gravity anomalies C(P,Q) given by Tscherning-Rapp can be defined as

$$C(P,Q) = A\sum_{NN+1}^{\infty}\frac{n-1}{(n-2)(n+B)}\, s^{n+2}P_n(\cos\psi) \qquad (6)$$

Modeling the covariance function means in practice fitting the empirically determined covariance function (through its three essential parameters; the variance C_0, the correlation length ξ and the variance of the horizontal gradient G_0) to the covariance function model. Hence the four parameters A, B, NN and s are to be determined through this fitting procedure. A simple fitting of the empirical covariance function was done using COVAXN-Subroutine (13).

The essential parameters of the empirical covariance parameters for 2489 gravity stations in Austria are 740.47 mgal^2 for the variance C_0 and 43.5 km for the correlation length ψ_1. The value of the variance for the horizontal gradient G_0 was roughly estimated as 100 E^2.

With a fixed value B = 24, the following Tscherning-Rapp covariance function model parameters were fitted: s = 0.997065, A = 746.002 mgal^2 and NN = 76. The parameters were used for the astrogeodetic, the gravimetric as well as the combined geoid solution.

3.3. Astrogeodetic Solution

The astrogeodetic solution by collocation is based on 659 deflections of the vertical uniformly distributed over Austria. After removing the long and short wavelength effects of the gravitational potential from the observations a geoid was estimated by LSC. Figure 2 shows the difference between the geoid solution by LSC and the GPS/leveling derived geoid. Be aware that the contour plot is based on the differences given at few selected GPS/leveling points. For the moment we are only interested in the long wavelength character of the differences which show a west-east trend of 1 m.

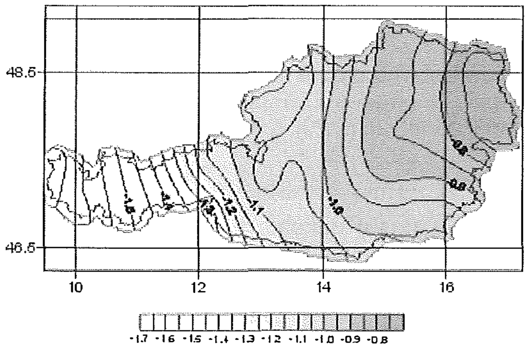


Fig. 2: Difference in geoid heights given in m, for the astrogeodetic geoid solution and the GPS/leveling geoid. Contour interval = 5 cm.

3.4. Gravimetric Solution

The gravimetric solution by collocation is based on the gravity anomaly data set mentioned in chapter 2.2 and presented in [4]. Figure 3 shows the difference between the LSC geoid solution and the GPS/leveling derived geoid. Once again the contour plot is based on the differences given at few selected GPS/leveling points. The differences are of the same magnitude as for the astrogeodetic geoid result (see Fig. 2). The differences show a high order polynomial trend with a west-east gradient of about 0.8 m over 600 km.

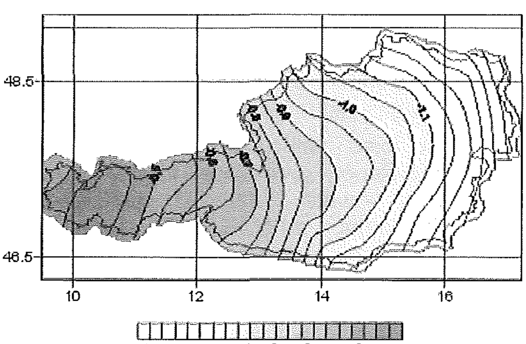


Fig. 3: Difference in geoid heights given in m, for the gravimetric geoid solution and the GPS/leveling geoid. Contour interval = 5 cm.

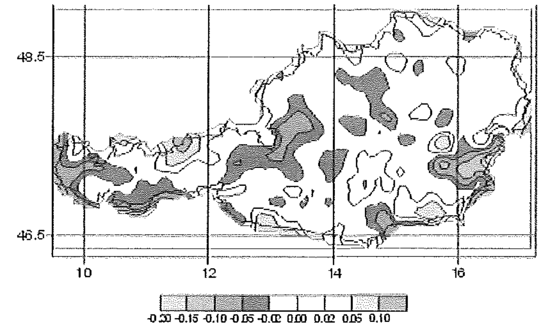


Fig. 4: Difference of the gravimetric solution fitted to GPS/leveling points minus the astrogeodetic solution fitted to GPS/leveling points. Difference given in cm.

Of particular interest is the comparison of the gravimetric solution and astrogeodetic solution. The differences are in most places less than ± 10 centimeters (see Fig. 4). One should notice that unequally spaced contour intervals were chosen to point out three different categories, namely ± 2 cm, ± 5 cm and ± 10 cm. The white pattern shows all points where the two solutions agree within ± 2 cm. Around 50% of the points fulfill this criterion. If we choose the category ± 5 cm, around 75% of the differences are within this range. A closer look at the largest differences reveals that no correlation with the topography exists. For instance the big differences along the Austrian border reflect the fact that the astrogeodetic solution is based on deflections points inside Austria only, while the gravimetric solution was computed on a more regional basis. The biggest difference is located in the eastern part of Austria.

We conclude that the difference probably depends on the distribution of the deflections of the vertical. The more homogeneous and dense the distribution of these points, the better the agreement between the two solutions. Of course, differences also depend on the smoothness of the residual gravitational potential. Therefore the residual gravitational field should be as smooth as possible and the used measurements should homogeneously cover the region in order to get a precise geoid solution.

3.5. Combined Solution

The first combined solution was done by Kühtreiber [14]. This combination of the gravimetric and astrogeodetic geoid was done by computing a simple arithmetic mean of the astrogeodetic and the gravimetric solution. In order to take the advantage of collocation as a method for combining gravity anomalies and deflections of the vertical in one estimation process, investigations concerning the relative weighting of these two data types are needed.

An important point concerning a reasonable combination of deflections of the vertical and gravity anomalies in a collocation process is the choice of the standard deviation for the different data types. A case study was carried out where the geoid heights were computed by a combination of deflections of the vertical and gravity anomalies. Three different cases, each with a different standard deviation for Δg but a fixed standard deviation for the deflections of the vertical, are considered. Each of the combined solutions is compared to the pure astrogeodetic and the pure gravimetric geoid solution (see Fig. 5).

Let us consider the starting configuration. The standard deviation for Δg was given as ± 0.3 mgal while the standard deviation for the deflections ξ and η were given as $\pm 0.2''$ and $\pm 0.3''$ respectively. Comparing the combined solution with the astrogeodetic solution shows more or less a big west-east trend with regions which don't fit the trend at all (e.g. eastern part of Austria). The difference between the combined solution and the gravimetric solution is a pure trend, no deviation from the trend is visible. The conclusions we can draw from the first pair of plots in Fig. 5 are:

- the standard deviation of ± 0.3 mgal for the gravity anomalies is too small. Hence the deflections of the vertical don't contribute to the solution on a local basis,

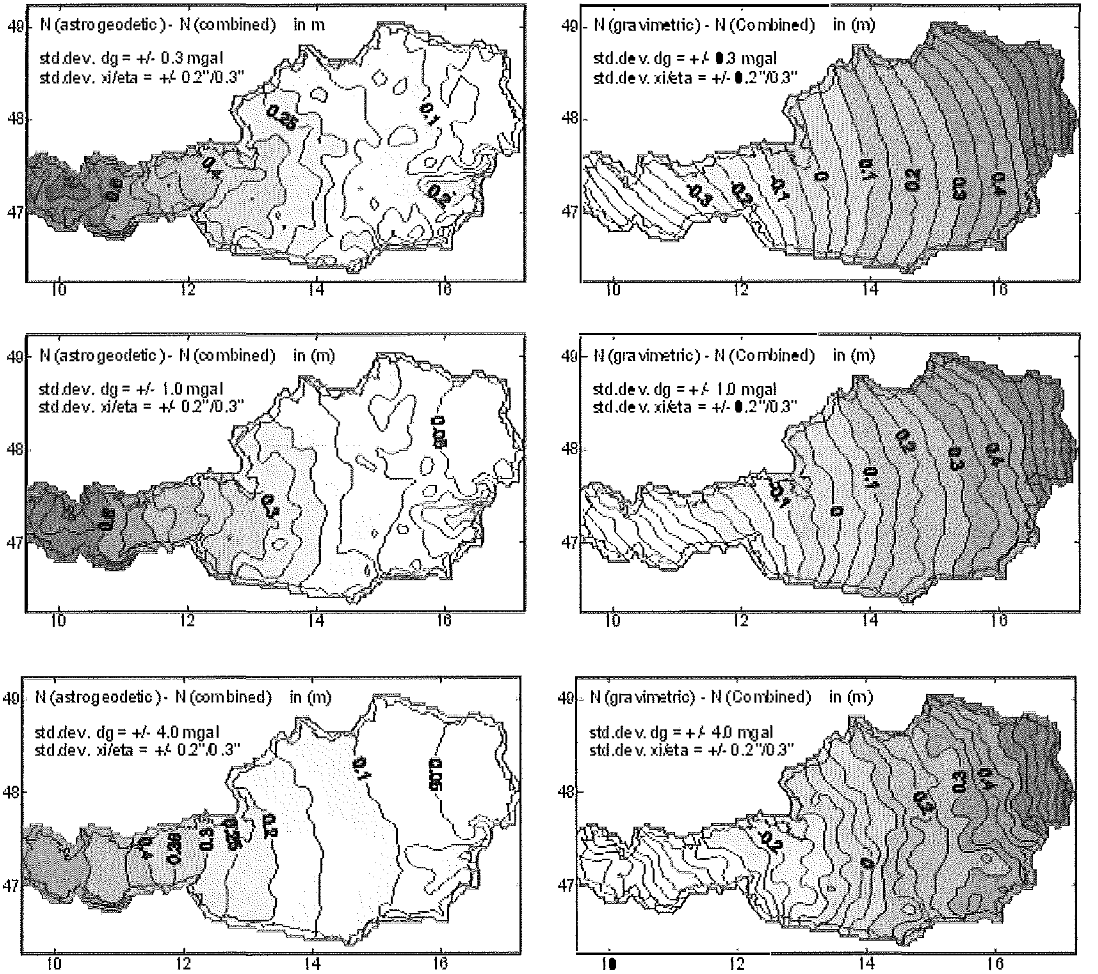


Fig. 5: Changing the standard deviation of the gravity anomalies in the combined solution and comparing the solution to the astrogeodetic and gravimetric solution.

- big discrepancies between the astrogeodetic and the combined solutions appear in regions with sparsely distributed deflections.

Increasing the standard deviation of the gravity anomalies and keeping the standard deviation of the deflections of the vertical fixed, should down-weight the influence of the gravity anomalies in the combined solution. Thereby the difference between the astrogeodetic and the combined solution should become a more or less pure regional trend, while the difference between the gravimetric and the combined solutions should become more irregular. The second and third pairs of plots in Fig. 5 prove this fact.

The poor results of the astrogeodetic solution in the east of Styria exist in all solutions. Even a very high weight for the deflections of the vertical relative to the gravity anomalies preserves the structure of the gravimetric geoid solution to some extent. This is clear as the deflections of the vertical are too sparse in this area to contribute to the combined solution.

As a conclusion of this study, the standard deviation of the gravity anomalies and the deflections of the vertical were chosen as ± 1.5 mgal for Δg and $\pm 0.2''$, $\pm 0.3''$ for ξ, η, respectively.

4. Comparisons with an extended GPS/leveling information

The comparison between the enlarged GPS/leveling set presented in chapter 2.4 and the geoid solutions of chapter 3 was performed in the following steps:

- *Interpolation of geoid undulations* (astrogeodetic, gravimetric and combined) for the available 102 GPS/leveling points by the use of Newton's interpolation algorithm for a regular grid. As a basis for the interpolation in each case 2847 grid points with a spacing of 3′ x 5′ were used. An individual point was calculated in the frame of the 16 adjacent grid points (degree 2 of Newton's interpolation).
- *Calculation of ellipsoidal heights* using the results of the above mentioned interpolation and leveled orthometric heights (chapter 2.5).
- *Calculation of 3D Cartesian coordinates* in combination with ETRF89-values (ϕ,λ) for each GPS-point.
- *Transformation into a best fitting position to the „real" GPS-derived ETRF89-values* by use of a 7-parameter Helmert transformation. The result of this transformation can be characterized by the following statistics:

solution	residual (mean value/cm)
ASTRO	4,0
GRAV	4,5
KOMB	3,7

- *Modeling the residuals:* The resulting residuals were modeled by use of Surfer 32 (Kriging, Point Griding in a regular grid of 10km x 10km) for the three given geoid solutions. (Fig. 6,7,8)

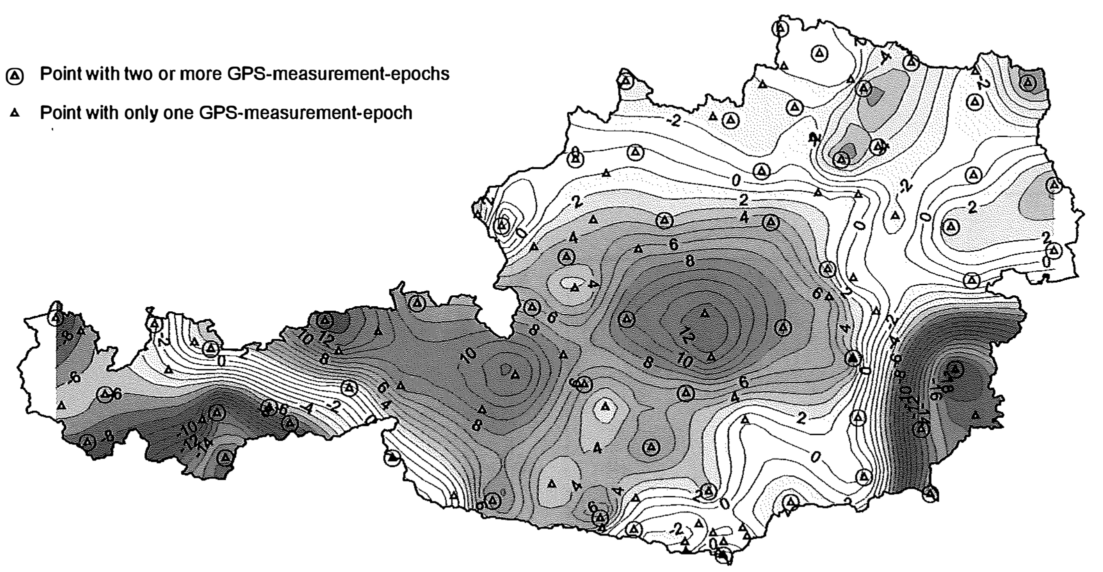


Fig. 6: Residuals GPS/Lev. minus ASTRO [cm]

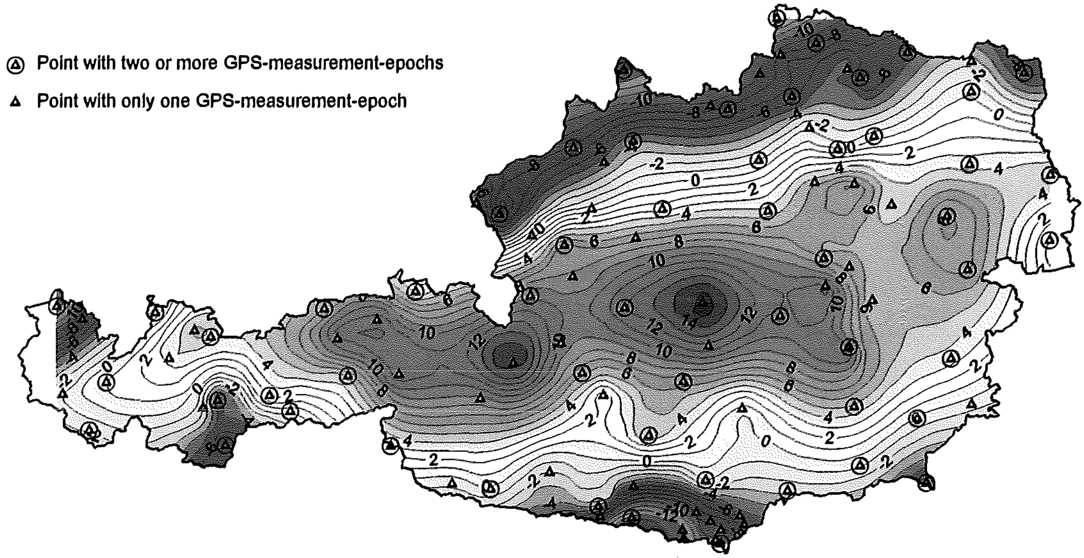


Fig. 7: Residuals GPS/Lev. minus GRAV [cm]

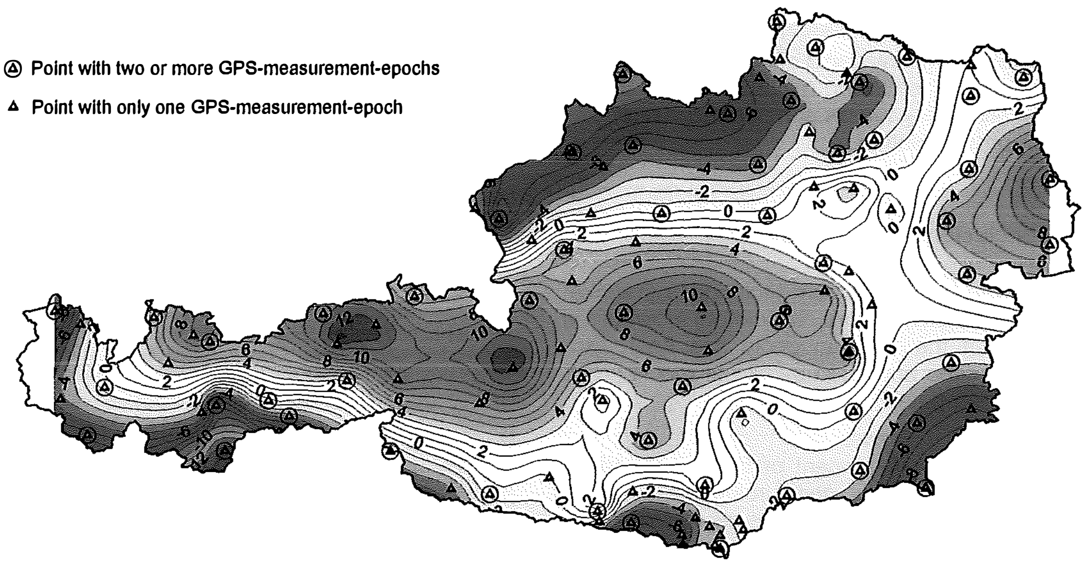


Fig. 8: Residuals GPS/Lev. minus COMBINATION [cm]

Generally the residuals show smooth long-wavelength distortions especially in the gravimetric solution. Again the residuals in the eastern part of the astrogeodetic solution are bigger than the overall trend. Further investigations in this area are needed.

5. The refined version of the Austrian geoid

By using and combining the astrogeodetic, the gravimetric data and the comprehensive GPS/leveling information presented above, a new refined version of the Austrian geoid could be performed. For this calculation the high precision of the available GPS-derived ellipsoidal heights as well as the high quality of orthometric heights recalculated within the new Austrian height system have to be taken into account.

Therefore the new geoid is based on the „Combined Solution" (chapter 3.5) and fitted to ETRS89 by use of the GPS/leveling information supplemented by the modeled residuals presented in Fig. 8.

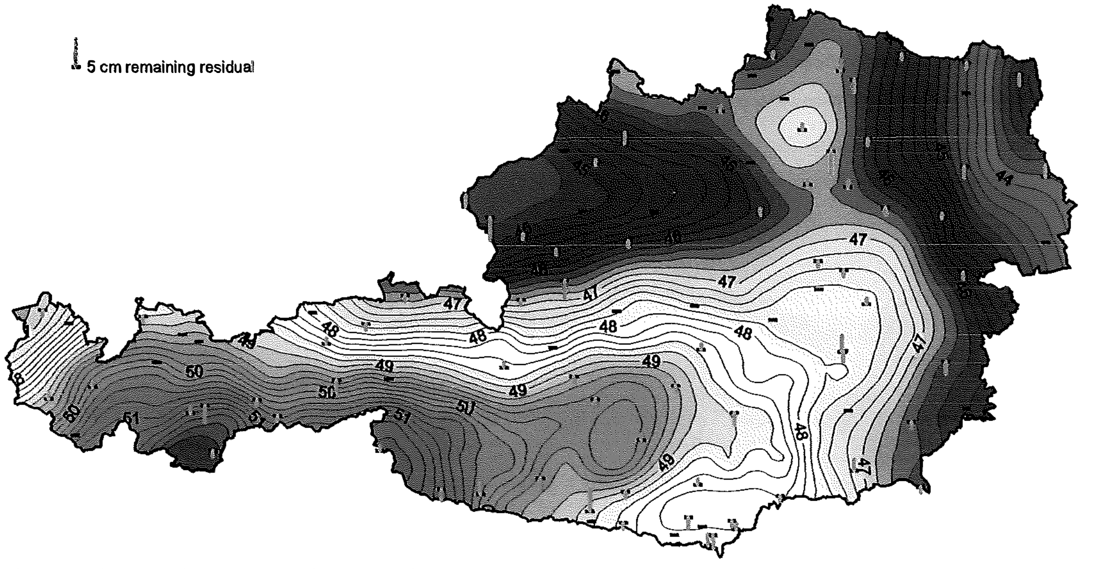


Fig. 9: Refined geoid after modeling the residuals

The resulting new geoid of Austria is presented in Fig. 9. The remaining residuals decrease to a mean value of $\pm 1{,}4$ cm and mirror the high precision of the solution.

6. Conclusions

With the help of modern techniques (remove-restore and least square collocation) a combined solution of the Austrian geoid has been estimated, refined and fitted by use of precise GPS and leveling data.

The solution shows:

- an excellent agreement of the astrogeodetic and the gravimetric geoid solution
- a centimeter precision of the geoid solution as a result of the combined version,
- the influence of high precision GPS- and leveling data in the fitting procedure and in the possibility of modeling long wavelength distortions.

Finally it must be pointed out, that the new Austrian geoid solution is a wide step forward, but there are still several remaining problems, which have to be investigated. One of these problems is the local distorsion of the astrogeodetic solution in the eastern part of Austria. At least a remeasurement and densification of the deflections of the vertical especially in that region together with additional GPS-leveling points covering the total area of Austria is planned.

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Contact

Dr. Erhard Erker, Dipl.-Ing. Norbert Höggerl, Dipl.-Ing. Erich Imrek: Bundesamt für Eich- und Vermessungswesen, Grundlagenvermessung, Schiffamtsgasse 1–3, A-1025 Wien. email: erhard.erker@bev.gv.at, norbert.hoeggerl@bev.gv.at, erich.imrek@bev.gv.at
Univ.-Prof. Dr. Bernhard Hofmann-Wellenhof, Univ.-Ass. Dr. Norbert Kühtreiber: Technische Universität Graz, Abteilung für Positionierung und Navigation, Steyrergasse 30, A-8010 Graz.
email: hofmann-wellenhof@tugraz.at, kueh@phgg.tu-graz.ac.at