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Results of Modelling GPS Satellite Clocks

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Results of Modelling GPS Satellite Clocks

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Abstract

The IGS (International GPS Service) Analysis Centers (ACs) provide GPS satellite clock offsets to GPS-Time (GPST) in the form of standard ephemeris in sp3-format or clock-RINEX files on a daily basis. These clock offsets, used mainly in GPS post-processing software along with consistent precise satellite ephemeris, are output to a least squares estimation process based on tracking data of the global IGS network.

Besides, to serve real-time applications, ACs have to forecast orbits and clock behaviour over a limited time span. The clock prediction models in use differ considerable both in terms of degree of the underlying polynomial as well as in the amount of observation data which enters a priori to fit the polynomial coefficients. First, we investigate the quality of the submitted clock offsets with respect to (w.r.t.) the observed combined IGS Rapid solution. Second, based on the satellites' clock-type specific behaviour, we try to set up a new model and to explore the stability and expected prediction errors of our approach.

Zusammenfassung

Für die Positionsbestimmung und Zeitübertragungsaufgaben mittels GPS benötigt der Nutzer Informationen über die Satellitenbahnen und -uhren. Die Analysis Centers (ACs) des IGS (International GPS Service) stellen die Bahnkoordinaten sowie die Abweichungen der GPS Satellitenuhren zu GPS-Zeit im sp3-Format zur Verfügung. Diese Files sind jeweils am folgenden Tag über einen freien ftp-Server erhältlich. Die Bahnen- und Uhren-Offsets sind das Ergebnis einer Parameterschätzung (vermittelnder Ausgleich nach der Methode der kleinsten Quadrate) auf Basis der Beobachtungsdaten des IGS-Stationsnetzes.

Für Echtzeit- oder beinahe Echtzeit-Anwendungen ist es notwendig, die Satellitenbahnen und -uhren für einen begrenzten Zeitraum vorauszuberechen. Die ACs verwenden für diese Prädiktion der Uhren verschiedene mathematische Modelle, die sich sowohl im Grad des Basispolynoms als auch im Umfang der verwendeten Eingangsdaten beträchtlich unterscheiden. In einem ersten Schritt beurteilen wir mittels einer groben Abschätzung die Qualität dieser Uhren-Offsets. Als Vergleichsdaten werden die Rapid Lösungen des IGS verwendet. Später wird versucht, ein eigenes verbessertes Modell für die Uhren-Prädiktion zu entwickeln. Nach der Bestimmung der Parameter eines Basispolynoms 2. Grades und gegebenenfalls einer additiven Sinusschwingung gelingt es schließlich, die GPS Satellitenuhren für 12 Stunden mit einer Genauigkeit von besser als ı2 ns vorherzusagen, was einem radialen Distanzfehler von ca. 50 cm entspricht.

1. Introduction

Precise point determination by means of GPS relies to a considerable extent on the quality of available satellite orbits and clock offsets with respect to GPST. GPST itself differs from the International Coordinated Time (UTC) by an amount of up to 40 ns (nanoseconds), ignoring leap-second differences. For many geodetic applications using differencing schemes the broadcast ephemeris, currently issued with an accuracy of about ±2 m and a corresponding clock rms of ±5 ns, are sufficient. To achieve highest precision, especially over baseline lengths larger than 10 km, the user is well-advised to take advantage of precise ephemeris provided by the IGS, [5]. Moreover, when analysing pseudorange and phase data, these products allow users to determine consistent coordinates and clock values even for an isolated GPS receiver with an internal accuracy of a few centimetres (precise point positioning), [4].

The IGS provides so-called 'IGS Final Orbits' of all GPS satellites on a weekly basis since the end of 1993. Compared to broadcast orbit information these ephemeris are more accurate by a factor of about 100, i.e. a few centimetres with a satellite clock rms of less than ±0.1 ns. They are available for post processing applications with a delay of 13 days (counted from the last day of the week which is contained in the orbit-file). IGS combined clocks are based on a linear alignment to GPST separately for each day. So while the internal stability of ±0.1 ns is quite good, the day-to-day stability of this reference is poor. Besides, the IGS also provides so-called 'Rapid' solutions with a slightly lower quality which are available at 17.00 UTC the following day. IGS Final and Rapid solutions are available from the IGS website at [5] free of charge.

It is worth mentioning that a joint project of the IGS and the BIPM (Bureau International des Poids et Mesures) aims to develop and demonstrate the operational capabilities of satellite navigation systems for time transfer, [1], [2]. In this context concepts to steer the IGS time scale to UTC were investigated, [3]. This will allow for a general dissemination of accurate and easily accessible UTC in the near future.

Moreover, since November 2000, the IGS distributes 'Ultra-Rapid' products (IGU) comprising precise GPS satellite orbits and satellite clocks for real-time or near real-time applications. This solution, issued twice daily, contains both an observed and a predicted part. Both cover a period of 24 hours. While the orbits of the predicted part are output to an integration of the well-known force field the clocks have to be extrapolated by means of a sophisticated prediction model.

2. Ultra-Rapid clock solutions

Official IGS products are the result of a weighted averaging process, based on individual submissions of up to 8 IGS Analysis Centres. This statement applies for both, IGU orbits and clocks. In the first step we are interested in a rough estimate of the overall quality of the individual ACs clock submissions to the IGS Ultra-Rapid combination. Thus we start with a raw comparison of the observed and the predicted clock-offsets w.r.t. the combined IGS Rapid clock solution which serves as reference in all calculations. Later on we will extract some statistical information from these clock-differences. Our calculations are based on the clock information given in the sp3-product files with a time resolution of 15 minutes. This kind of comparisons are carried out at the Department of Advanced Geodesy (TU Vienna) since GPS week 1151, February 2002 and reported regularly at [7].

Raw clock differences usually reflect the clock offset and the clock drift of reference clock 2 w.r.t. reference clock 1 (see figure 1). In contrast to the combined IGS Rapid clock product (linearly aligned to GPS-time) the reference clocks used in AC solutions are steered to a very stable clock at one of the tracking sites or to a weighted assembly of hydrogen masers located in timing laboratories around the world. A clock-offset and the clock-drift are common to all reported satellite clocks. In addition clock-differences may reflect radial orbit differences (per satellite) of the corresponding ephemeris, which propagate into the clock solutions. For the observed 24 hours part these differences induced by the

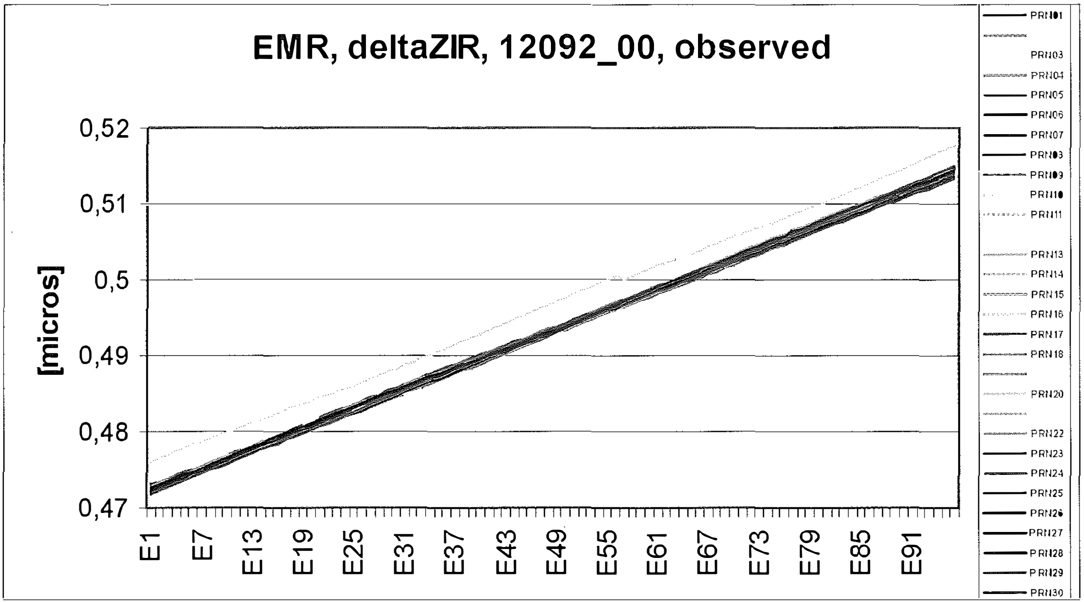

Figure 1: EMR satellite clock solution w.r.t. combined IGS Rapid clocks / GPS-week 1209, day 2. EMR ... Natural Resources Canada, Ottawa

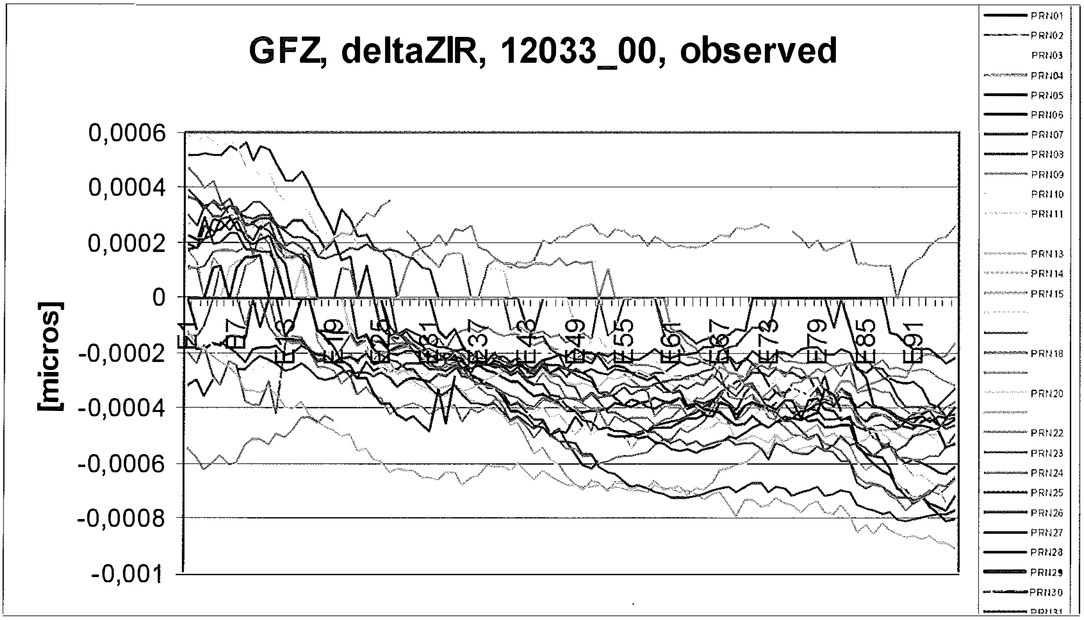

Figure 2: GFZ satellite clock solution w.r.t. combined IGS Rapid clocks / GPS-week 1203, day 3.
GFZ ... Geoforschungszentrum, Potsdam

orbits usually range up to a few tenth of a nanosecond (1 ns = 30 cm), see figure 2. In all diagrams of this paper label 'ZIR' stands for the IGS Rapid solution. Due to the 15-minutes binning, the time axis covers 96 epochs during a day (E1-E96).

Figure 3 demonstrates a common jump of about 1 ns around epoch E30. The graphic points to a phase-jump of the reference clock used in the ESA solution whereas a reset of an individual satellite clock would affect only that particular satellite. Phase jumps of the reference

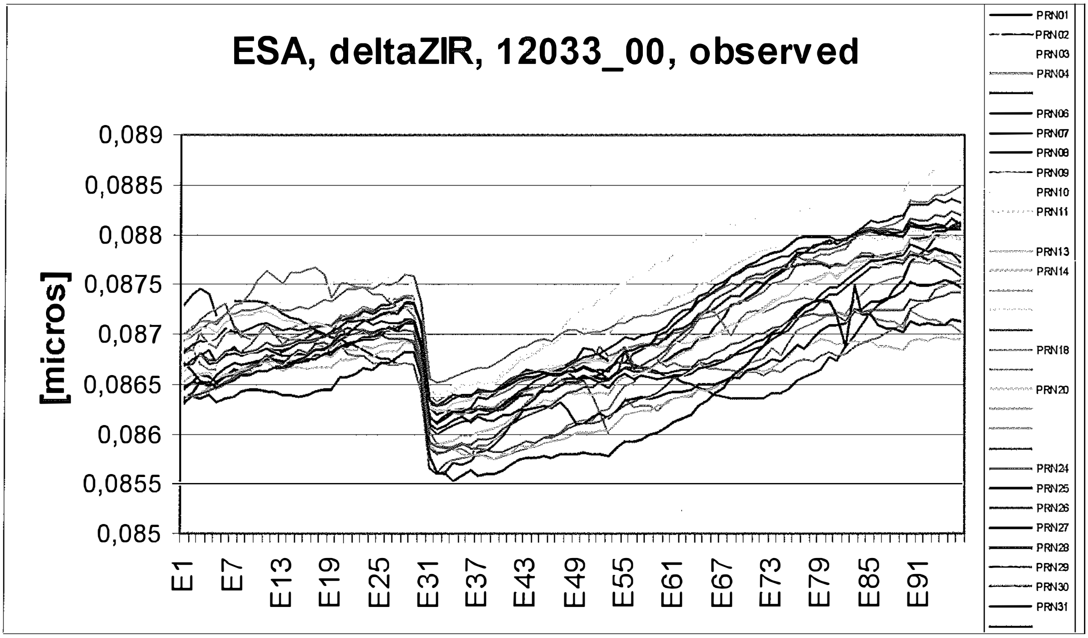

Figure 3: ESA satellite clock solution w.r.t. combined IGS Rapid clocks / GPS-week 1203, day 3.
ESA ... European Space Agency, Darmstadt

clock can be rather large, e.g., up to a couple of microseconds.

If the reference clock fails, the AC might switch to another reference clock of compatible quality. Currently about 30 stations within the IGS network are equipped with hydrogen masers and a few of them provide a stability of better than 2x10^{-15} for one day. It might be of interest to the reader that switching to another reference clock does not harm positioning as far as all satellite clock-offsets to GPS-time given for a single epoch refer to the same reference oscillator.

When inspecting the 24 hours period of clock prediction we find a complete different scenario. While the clock-differences of the observed part normally populate a small band of 1–2 ns, the values within the predicted part diverge substantially, see figure 4. Another outcome of the diagram is, that obviously some satellite clocks are more difficult to predict then others. Usually clock predictions over 12 hours are good to ±3–5 ns, but depending on the stability of the satellite clock the prediction might be wrong by 10 ns over the same period. Figures 4a,b cover the same time slot in GPS-week 1204. Two IGS centres, namely USN (US Naval Observatory, Washington) and again ESA tried to predict the satellite clock behaviour during that period. Obviously the centres obtain their results from different prediction models. Ignoring a common drift and offset w.r.t. the reference clock (ZIR), which is insignificant for positioning, the USN predictions diverge after 24 hours by about 70 ns (satellites 10, 27) while the ESA predictions diverge by 60 ns.

In a second step we calculate the clock rms of the offset and drift reduced clock differences both for the observed and the predicted part. In detail, the ACs clocks are reduced for a satellite individual offset and drift w.r.t. to the combined IGS Rapid clocks. Thus, the remaining residuals Δ ZIR reflect solely periodical deviations which leads of course to very optimistic rms-estimates:

$$\mathrm{rms}_i = \sqrt{(\sum(\Delta\,\mathrm{ZIR}_i)^2)/n} \qquad (1)$$

with ΔZIR .. difference between the reduced ACs Ultra-Rapid clock solution and the IGS Rapid clock solution
i .... satellite
n .... number of epochs

As demonstrated in figure 5 the rms of observed satellite clocks typically range from 0.1 ns to about 0.4 ns. This result might be a little disappointing when compared to IGS Final or IGS Rapid clocks which are of a higher quality by a factor of 3. However, we should keep in mind that Ultra Rapid products are based on a relatively small quantity of immediately available tracking data.

For the predicted part, the clock rms is calculated in different intervals as shown in figures 6a,b. The intervals start at 0.00 GPST with the first predicted clock offset and last for 3, 6, 9, 12, and 24 hours, respectively. Again the clock differences have been reduced for an offset and a drift in advance. The satellite specific clock

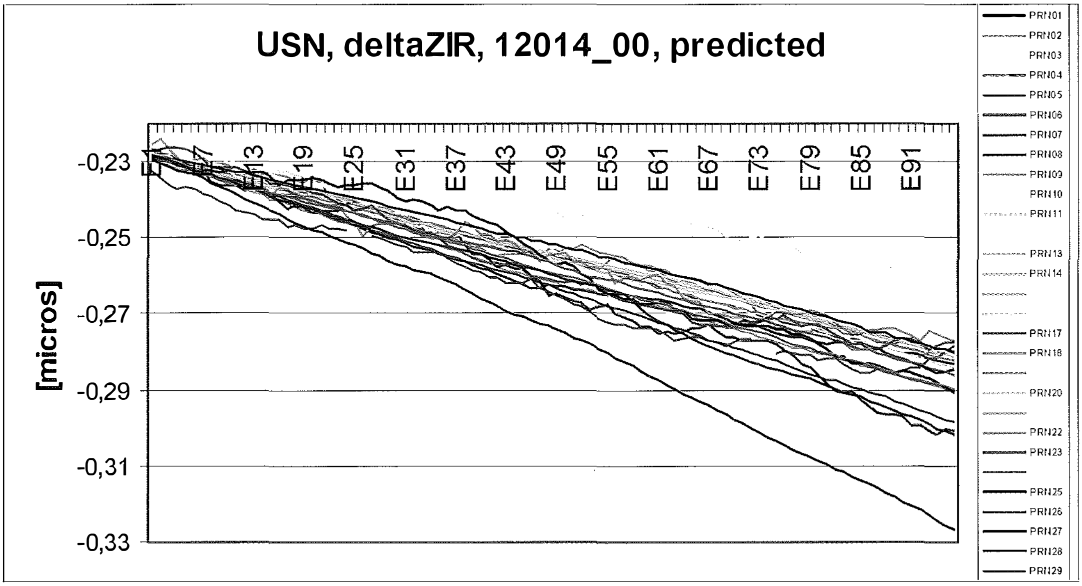

Figure 4a: USN satellite clock prediction w.r.t. combined IGS Rapid clocks / GPS-week 1201, day 4.

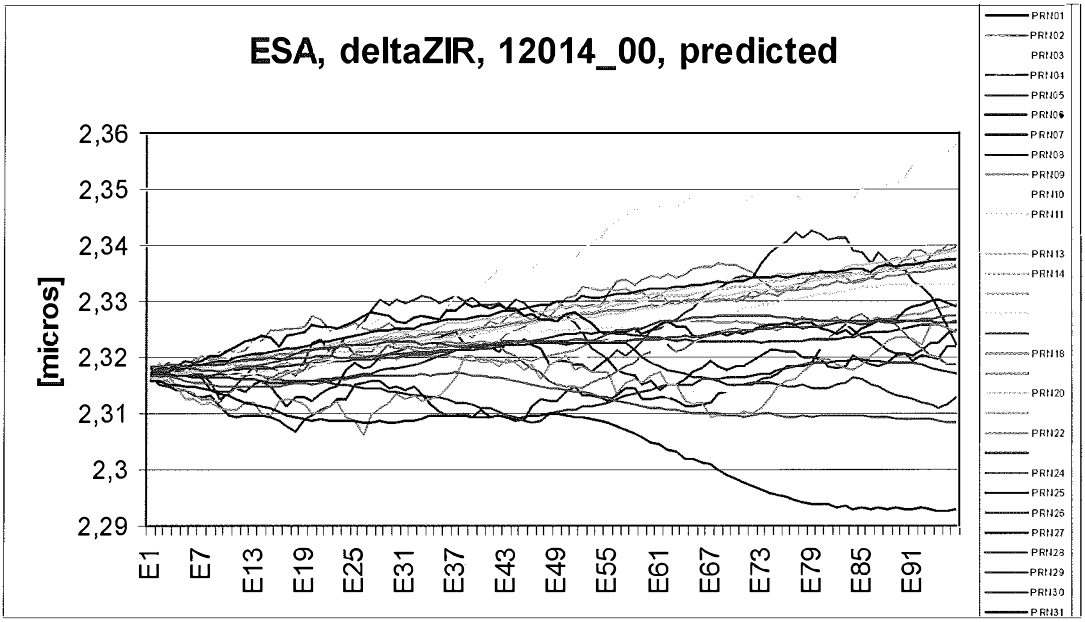

Figure 4b: ESA satellite clock prediction w.r.t. combined IGS Rapid clocks / GPS-week 1201, day 4.

rms for the predicted interval of 24 hours may reach 10 ns or more. As expected the rms-values increase in most cases with the length of the interval. A series of steady growing bars reflect a significant quadratic or periodic behaviour of the satellite clock (see figure 6a, PRN23). For comparison the AC-solutions presented in figures 6a,b coincide in time with figures 4a,b. Please note the different scale in figures 6a,b,c.

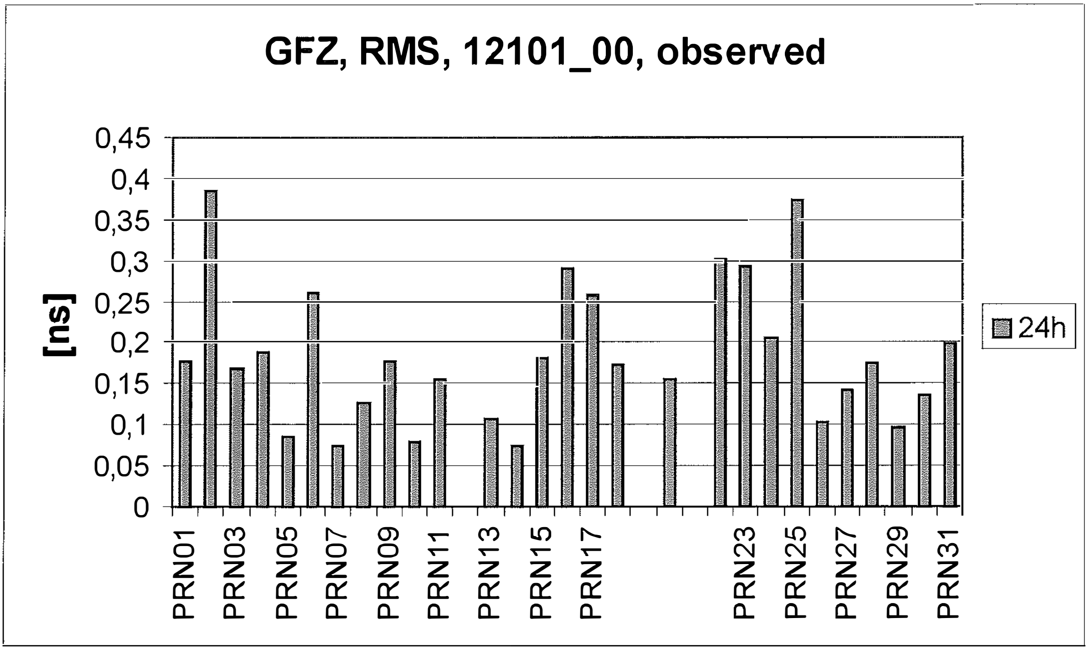

Figure 5: Satellite clock rms of GFZ observed Ultra-Rapid solution w.r.t. ZIR / GPS-week 1210, day 1.

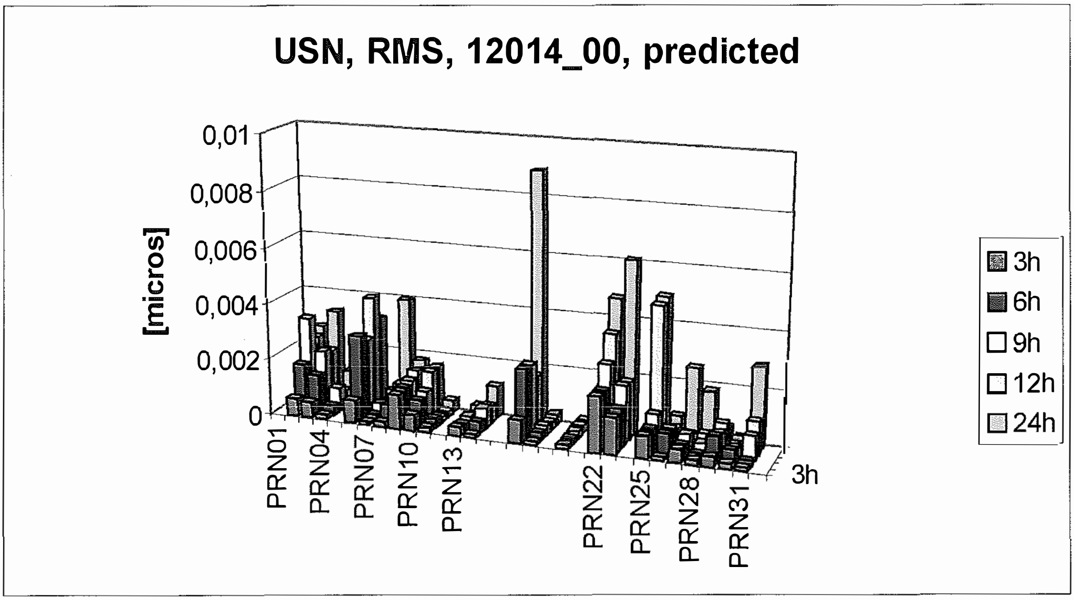

Figure 6a: Satellite clock rms of USN predicted Ultra-Rapid solution w.r.t. ZIR / GPS-week 1201, day 4.

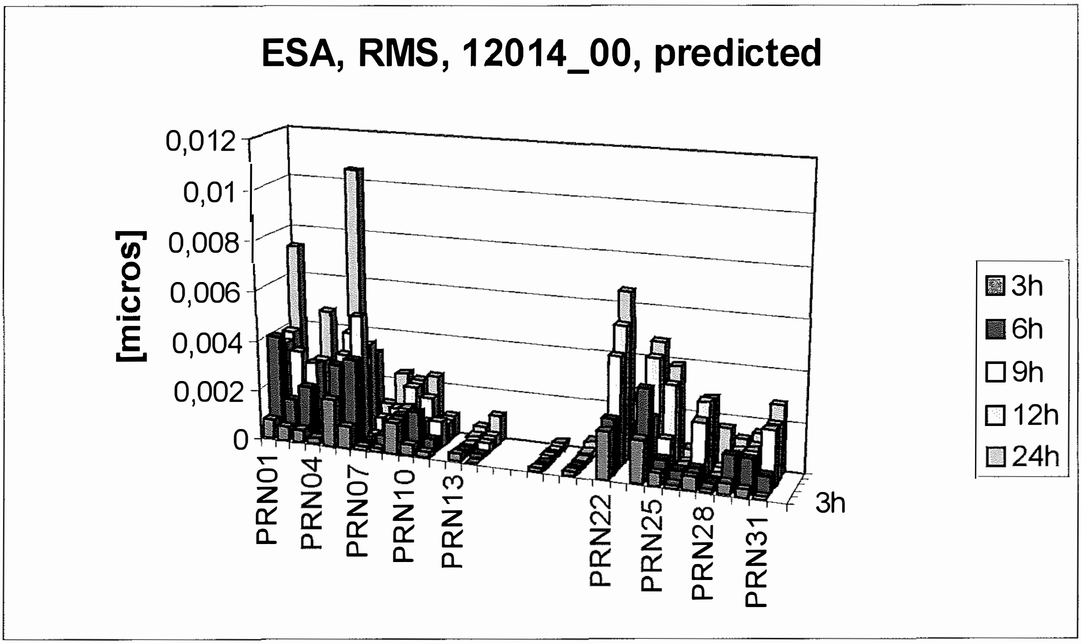

Figure 6b: Satellite clock rms of ESA predicted Ultra-Rapid solution w.r.t. ZIR / GPS-week 1201, day 4.

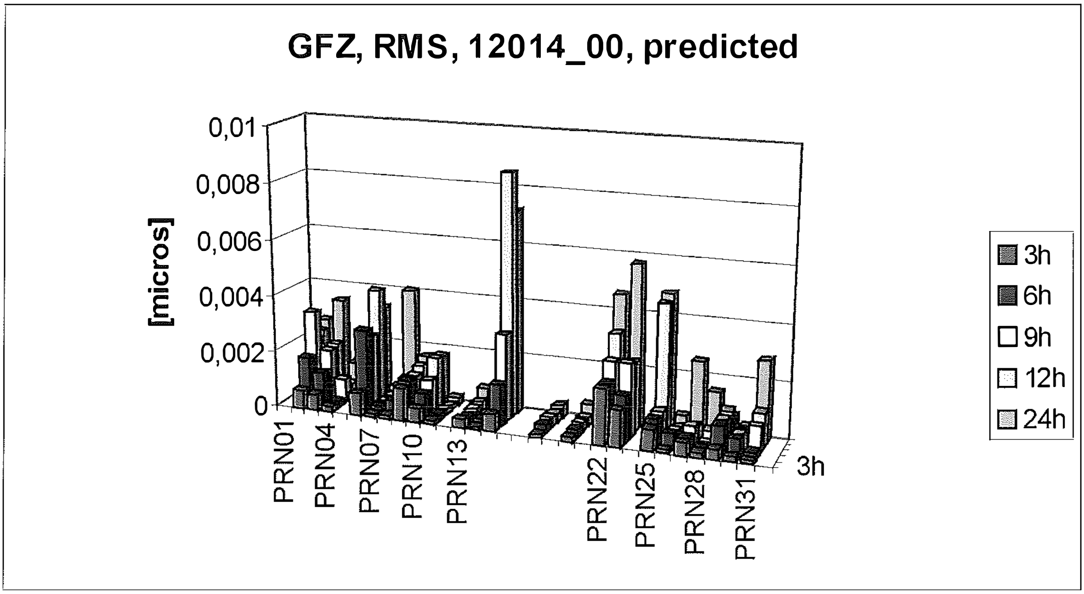

Figure 6c: Satellite clock rms of GFZ predicted Ultra-Rapid solution w.r.t. ZIR / GPS-week 1201, day 4.

3. Prediction of GPS satellite clocks

In order to explore reliable prediction models, valid over a span of 24 hours, we use the available observed part of the IGS Ultra-Rapid solutions of the past 48 hours. A least squares adjustment determines the coefficients of a polynomial of first or second order. Depending on clock type and behaviour (Cesium or Rubidium) we add cyclic terms.

To establish a continuous set of clock-offsets per satellite over the past 48 hours we have to deal with 'fictitious' clock jumps which show up in our sp3-files at the day boundaries. These jumps stem from the alignment of the IGS Rapid and Ultra-Rapid clock products to GPST. A very preliminary approach to bridge the gap in clock-offsets at the day boundaries is to shift all the data of the first 24 hours by a constant. Obviously a difference in drift still remains in the data. The resulting series serve as input for the determination of a polynomial to be used for clock prediction on the following day.

An interesting fact thereby is, that an expression valid for predicting a GPS satellite clock is obviously dependent on the clock-type of the individual satellite. The most recent GPS satellites (IIR) are equipped with Rubidium clocks, whereas others use Cesium clocks. The stability of a Cesium oscillator is about $1x10^{-14}$ over a day, whereas the stability for a Rubidium oscillator is limited to $1x10^{-12}$ over the same period, [9].

In table 1 the nature of the clock for each GPS satellite is shown, where "Rb" stands for Rubidium an "Cs" for Cesium:

PRN	CLOCK	PRN	CLOCK	PRN	CLOCK
01	Cs	11	Rb	23	Cs
02	Cs	13	Rb	24	Cs
03	Cs	14	Rb	25	Cs
04	Rb	15	Cs	26	Rb
05	Cs	16	Rb	27	Rb
06	Cs	17	Cs	28	Rb
07	Rb	18	Rb	29	Rb
08	Rb	20	Rb	30	Rb
09	Cs	21	Rb	31	Rb
10	Cs	22	Cs		

Table 1: Types of satellite clocks [8] as of April 2003

We analysed the drift reduced clock offsets to obtain reasonable apriori information about the behaviour of the individual GPS satellite clocks. From the remaining residuals we may deduce that the empirical formula describing the clock behaviour w.r.t. to a stable reference clock reads:

$$p(t) = a \cdot t^2 + b \cdot t + c + A_0 \cdot \sin(\omega \cdot t + \varphi) \qquad (2)$$

with a, b, c ...... coefficients of the polynomial
A_0 .... amplitude
ω .... frequency
φ .... phase shift

The argument t varies from 1–192 over the considered time span of 48 hours. We learn from figure 7a that a typical Rubidium clock shows no significant cyclic terms and can be sufficiently described by a simple quadratic polynomial.

For satellites with Cesium clocks the situation looks slightly different. In case of Cesium clocks it is reasonable to set up a linear polynomial (instead of a quadratic one) with an additional sine

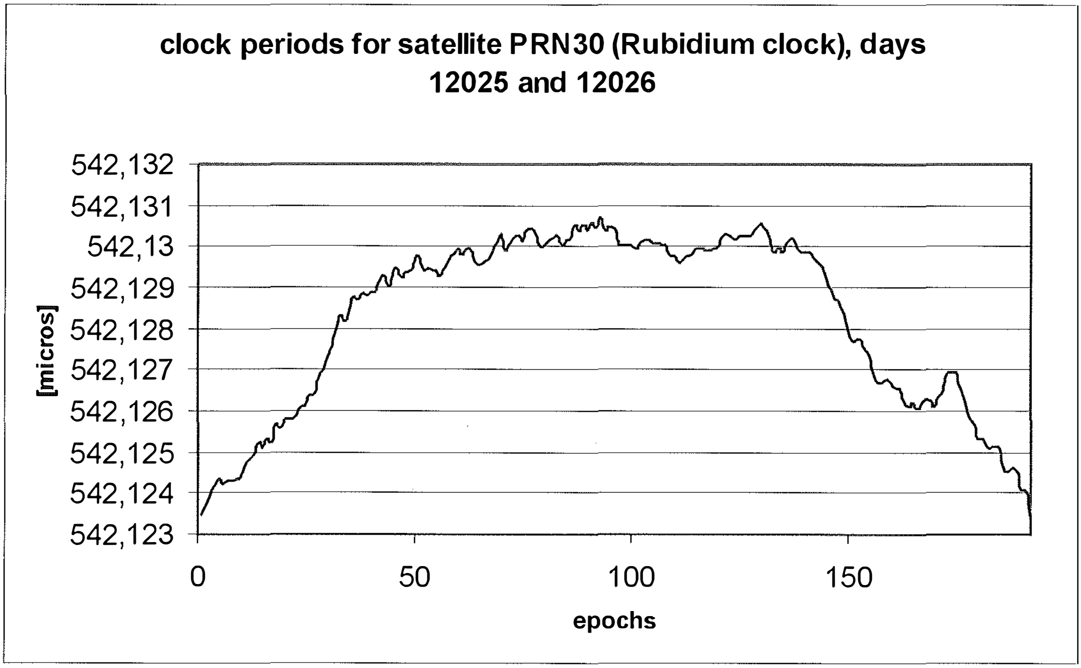

Figure 7a: Typical behaviour of a Rubidium clock over 48 hours / GPS-week 1202, days 5–6.

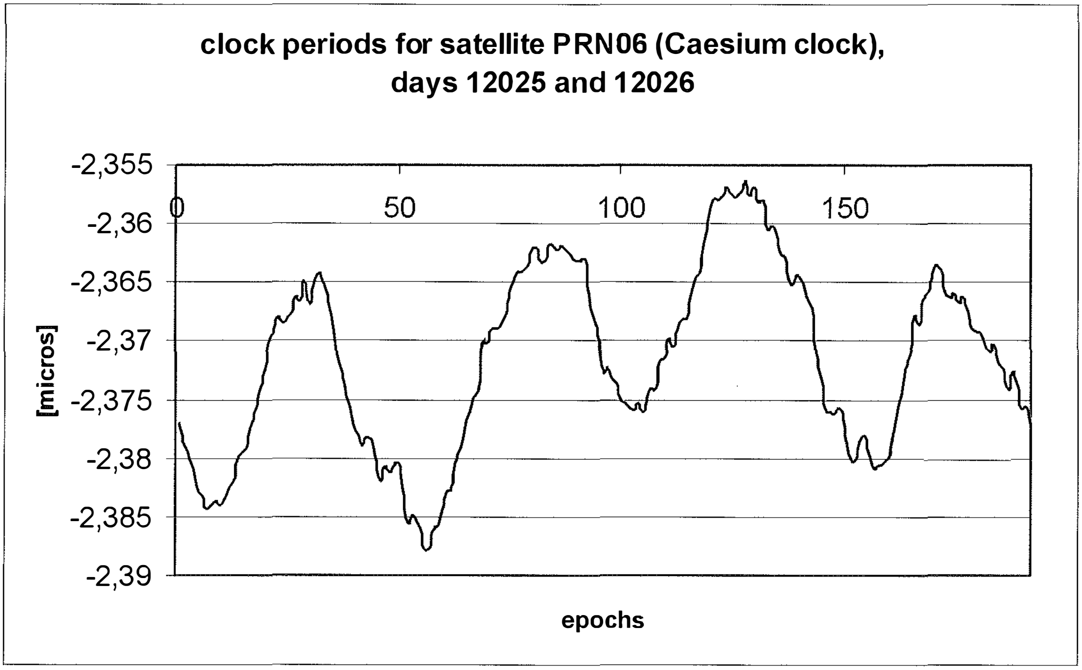

Figure 7b: Typical behaviour of a Cesium clock over 48 hours / GPS-week 1202, days 5–6.

term (see figure 7b). Within the determination of the sine term the amplitude and the phase shift to the input-data were fitted, whereas the main frequency was introduced as a predefined constant for each satellite. Most satellite clocks vary with a period of 12 hours which is close to the revolution period. These periods are not easy to de-correlate from orbit errors in the reference solution. We also found a couple of shorter, less significant, periods. A typical diagram of the variations of a Cesium satellite clock is shown in figure 7b. Both diagrams 7a,b cover 2 days.

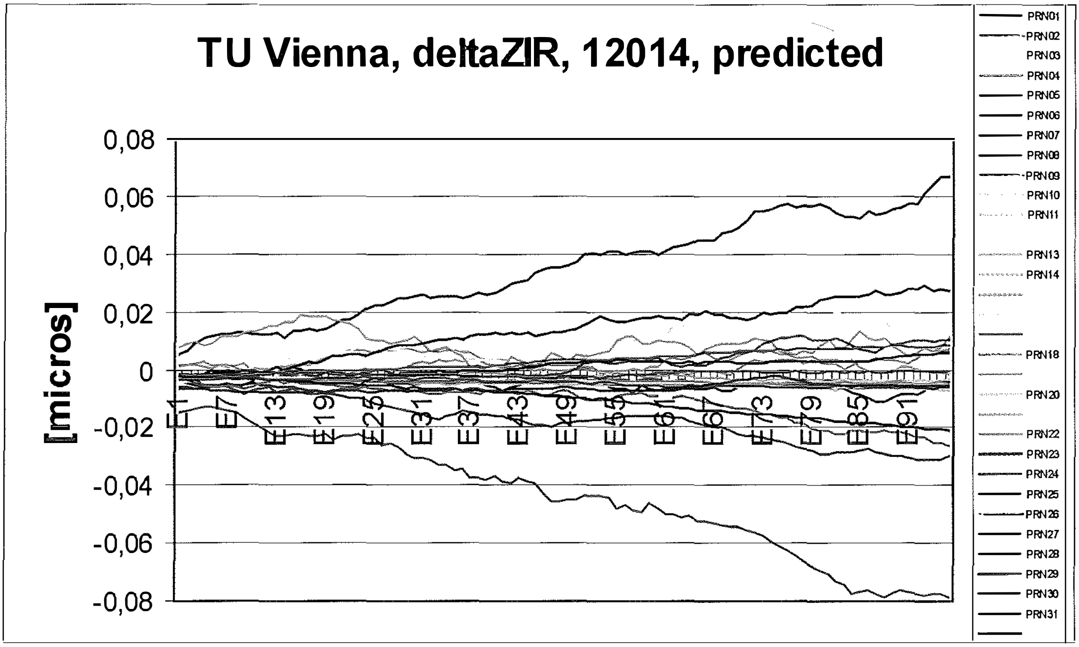

Figure 8: TU Vienna satellite clock prediction w.r.t. combined IGS Rapid clocks / GPS-week 1201, day 4.

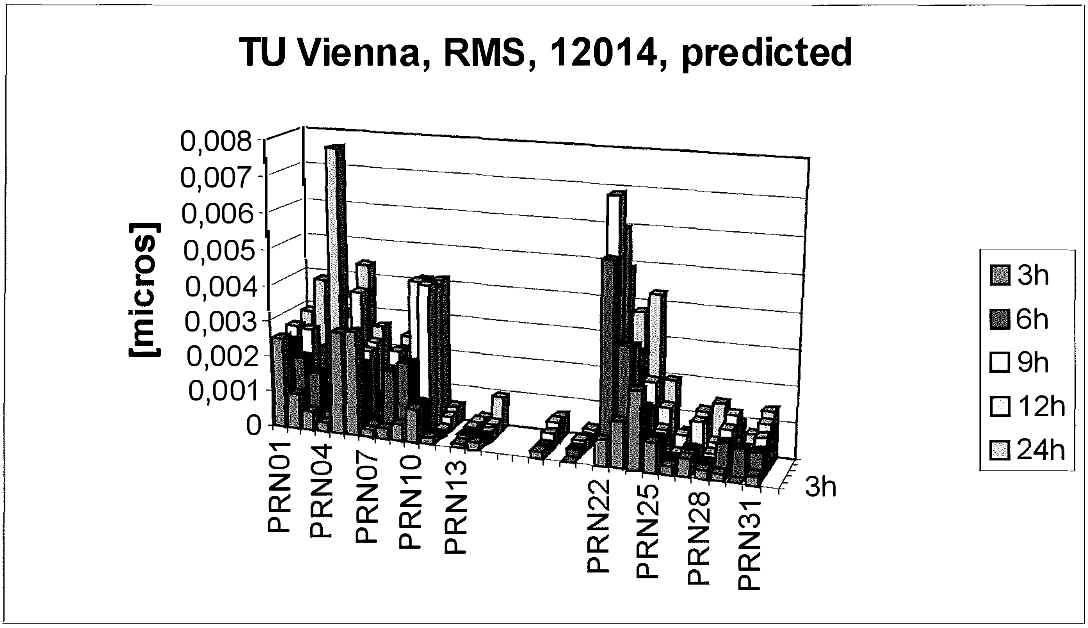

Figure 9: Satellite clock rms of TU Vienna predicted Ultra-Rapid solution w.r.t. ZIR / GPS-week 1201, day 4.

We may summarize that for satellites with no noticeable periods in the reduced clock-offsets solely the parameters of a quadratic function were estimated within a least squares adjustment. For those satellites which show a more complicated behaviour the parameters of a polynomial of first or second order and the amplitude and phase shift of an additional sine term were calculated. The coefficients for t^2 are in general rather small and in most cases significant. The amplitude of the cyclic term reaches up to a few nanoseconds. The phase shift is different for all satellites. The estimated functions are used to predict the satellite clocks on the following day. The results, labelled 'TU-Vienna', are compared with the IGS Rapid solutions for the same day (see figures 8 and 9). Satellite PRN15 is missing in our diagrams because no Ultra-Rapid solution was available for this satellite on days 2 and 3 of GPS-week 1201.

The results of our preliminary analysis are quite satisfying. However, figure 8 clearly indicates that further work has to be invested how to treat 'complicated' satellites like e.g. PRN01 and PRN05. Improvements are expected from dealing correctly with the different clock drifts in each 24 hours span of our input data. Also increasing the observation span for fitting reasonable forecast functions might be worth to investigate. Nevertheless the goal of providing clock predictions better than ±1 ns for the subsequent 3–6 hours for real time point positioning will be extremely hard to achieve.

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